

书名: Mathematical Analysis, Second Edition

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[书籍评论]

本书第一章以公理化的方式引入了实数系和复数系, 接下来介绍了集合论和点集拓扑的一些基本概念和内容, 为后面微积分理论的展开打好基础。从第四章开始, 作者开始介绍极限、连续和导数等微积分的基本概念。在第六章作者引入了有界变差函数与可求长曲线的概念, 接着就对 Riemann-Stieltjes 积分进行了介绍, 而 Riemann 积分则是它的特例。第八第九章是对级数和函数序列知识的讲解。第十章介绍 Lebesgue 积分, 第十一章介绍 Fourier 级数以及 Fourier 积分, 第十二章介绍多元微分学, 第十三章介绍隐函数与极值问题, 接下来的两章是关于多重 Riemann 积分与 Lebesgue 积分的介绍, 最后一章介绍了复变函数的 Cauchy 定理以及留数的计算。

本书是一部现代数学名著: 自 20 世纪 70 年代面世以来, 一直受到西方学术界、教育界的广泛推崇, 被许多知名大学指定为教材。作为一本大学数学系的本科教材, 本书仔细而又不累赘地向读者介绍了微积分的思想, 涵盖了数学分析绝大部分的基本知识点, 并配有覆盖各级难度的练习题, 适用于初次接触数学分析的读者。无论对于教学还是自学, 都不失为一本理想的教材。另一方面, 本书对于实分析和复分析中的部分内容也有所介绍, 这其实也是很多美国大学数学教材 (Mathematical Analysis 或者 Advanced Calculus) 内容设置的共同点。例如作者在第十章有对 Lebesgue 积分的介绍。不过与一般实分析教材里的思路不同, 作者采用了 Riesz-Nagy 的方法引入了 Lebesgue 积分, 此方法直接着眼于函数及其积分, 从而避免了对于测度论知识的要求; 同时作者还进行了简化、延伸和调整, 以适应大学本科水平的教学。

[作者简介]

Tom M. Apostol, 美国数学家, 生于犹他州。他于 1946 年在华盛顿大学西雅图分校获得数学硕士学位, 于 1948 年在加州大学伯克利分校获得数学博士学位, 1962 年起任加州理

工学院教授，美国数学会、美国科学发展协会（A.A.A.S）会员。对初等数论和解析数论有研究，他的著作很多，除本书外，还著有《Calculus, One-Variable Calculus with an Introduction to Linear Algebra》、《Calculus, Multi-Variable Calculus and Linear Algebra with Applications》等。

（徐晓津）

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