[书名]：Problems and Theorems in Analysis　分析中的问题与定理
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[出版商]：世界图书出版公司
[页数]：第 1 卷 389 页；第 2 卷 391 页；共 780 页
[适用范围]：数学专业高年级学生与研究生，数学教师与数学工作者
[预备知识]：数学分析，高等代数，复变函数
[习题数量]：第 1 卷 776 道；第 2 卷 884 道。这是一套习题书。
[习题难度]：难，有的习题甚至为研究者的最新成果，难度很大
[推荐强度]：9.8/10

[书籍评价]

本书两卷，共分九个部分。第一部分主要收录无限序列与无限级数方面的问题。第二部分是有关积分的各种问题。第三、第四部分是关于单复变量函数的问题，内容包含了数学系本科生与研究生的复分析课程中的主要问题。第五部分主要涉及代数的零点确定问题。第六部分讲多项式与三角多项式。第七部分为行列式与二次型的问题。第八部分为数论方面的题目。第九部分为数学中与几何有关的一些问题。

本书与其说这是一部教科书，不如说这是一部字典，因为它收录了分析学中的各种问题和定理。这是一本有着突破传统意义的书。它对问题巧妙的系统性安排与归纳，给学生创造了自主性思考的可能，最大程度上启发学生的研究能力和创新能力，这也是它不同于其他一些平庸的习题参考书的地方。作者甚至试图用很多哲学的观点来阐释它所选出的题目的代表性，比如有关特殊和一般的问题，要知道早期著名的数学家迪卡尔曾经说过：“我学数学是为了追求最终的哲学。”正是这种理念的融入，使得这本书在学术界的地位尤为突出，不只是学生，很多教授和数学工作者都以此书为参考书，并对此书给予了高度的好评。

[零星感悟]

什么是好的教育？给学生一套完善的体系然后让学生在这样的体系下寻找机会自己去发现和解决问题，这样的完善的体系才是好的教育的关键。此习题书不同于其他习题参考书的特点也就在此。它给我们数学系高年级学生与研究生提供了在不同主题下精心安排的问题，启发我们独立思考和研究问题的能力，是一本不可多得的分析习题书籍。

第一部分的习题 139 让我们明白了很多问题就像两个点决定一条直线一样，是有两个极端的线性组合而得出的结论。

第六部分的习题 92 让我们明白了掌握一个领域的知识就像了解一个城市的所有交通路线。真正的掌握就是从任何一个出发点，你都可以找到最短的路线达到你想要达到的目的。

[作者简介]

George Polya（1887-1985）匈牙利数学家，早年在苏黎世瑞士联邦理工学院任教，后入美国籍，1942 年起在美国 Stanford 大学任教。Polya 在数学的广阔领域里都有深入的研究，特别在泛函分析、数理统计和组合分析等方面尤为突出。Polya 不仅是数学家，也是一为优秀的教育家，他始终把高深的数学研究和数学的普及与教育结合起来。

Gabor Szego（1895-1985）匈牙利数学家，早年在柯尼斯堡大学任教，后入美国籍，也在美国 Stanford 大学任教。他主要的贡献是在数学分析与数理方程方面。

《分析中的问题与定理》一书是George Polya 与 Gabor Szego 最著名的著作。Polya曾经这样评论他与Szego的合作：这是一段美妙的时光；我们专心致志、充满热情地工作。我们有着同样的背景。我们象同时代其他匈牙利数学家一样，受到Leopold Fejér的影响。我们都是那个为中学生创办的强调解题的刊物Hungarian Mathematical Journal的读者。我们又对同样的课题、同样的问题感兴趣，但往往是一个人对某一个课题知道得多，而另一个人对其他

的课题知道得多。这是一次绝妙的合作。我们的合作成果—《分析中的问题与定理》，是我最好的工作，也是Szego最好的工作。

(刘东弟)

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